Titanic prime quintuplets

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As any chemist will tell you, the word 'titanic' usually means 'pertaining to the tetravalent state of titanium, Ti, the 22nd element in the periodic table.' However, as applied to prime numbers the adjective has a specific and entirely different meaning, coined by Samuel Yates in 1985.

> A *titanic* prime is defined as a prime number which has at least 1000 decimal digits.

This same definition is used by Chris Caldwell in his database of large primes at http://primes.utm.edu/.

The first titanic prime was discovered by Alexander Hurwitz in 1961. He had programmed the computer to search for Mersenne primes and left it running overnight. When he looked at the results he found two, namely $2^{4253} - 1$ and $2^{4423} - 1$. The story goes that because of the way computer printers of that era worked the output was presented in a back-to-front manner. So, as he flipped over the pages, Hurwitz would have seen the larger prime first. Thus it generally agreed that first titanic prime discovered by a human was

$$2^{4423} - 1 \quad \text{(1332 digits)}.$$

The *smallest* titanic prime, $10^{999} + 7$, was discovered many years later. Although for quite a long time everybody 'knew' that $10^{999} + 7$ was prime, it was not until 1998 that a proof (by Preda Mihailescu) appeared.

The first titanic twin primes were found in 1980 by Oliver Atkin and N. W. Rickert:

$$256200945 \cdot 2^{3426} \pm 1 \quad \text{(1040 digits)}.$$

During the 1990s I became interested in the subject and in December 1996, I was able to report the first proven titanic prime triplets [M500 **154**],

$$437850590\,(2^{3567} - 2^{1189}) - 6 \cdot 2^{1189} + d,\ d = -5,\ \ -1,\ \ +1 \quad \text{(1083digits)}.$$

With more powerful equipment I carried out another computer search in September 1998, which resulted in the discovery of the 1003-digit titanic quadruplets,

$$76912895956636885\,(2^{3279} - 2^{1093}) - 6 \cdot 2^{1093} + d,\ d = -7,\ \ -5,\ \ -1,\ 1$$

[*Math. Gazette*, November 2000].

The challenge to find a set of titanic prime *quintuplets* (i.e. five 1000-digit primes packed together as closely as possible) was taken up by **Norman Luhn**. As you can imagine, this was a formidable task. Even as I write, there exist only a handful of known titanic prime quadruplets (the largest having 1284 digits); but quintuplets of similar magnitude are much rarer, and one would be forgiven for dismissing as unfeasible a search for such objects using the current generation of personal computers.

Therefore I was most surprised when, on 30 July 2002, Norman informed me that he had discovered the first ever set of *titanic prime quintuplets*,

$$31969211688 \cdot 2400\# + 16061 + d,\ d = 0,\ 2,\ 6,\ 8,\ 12 \quad \text{(1034 digits)},$$

where $x\#$ denotes the product of all the primes not exceeding x. The first one (corresponding to $d = 0$) is written out in full on the front cover of this magazine [see page 3]. To save you counting, there are 20 lines of 50 digits and one line of 34; 1034 digits in all. The other four primes are the same except that they end in 0943, 0947, 0949 and 0953, respectively.

Some more prime number records, as at November 2002

Largest prime

$2^{13466917} - 1$ (4053946 digits), Michael Cameron, George Woltman, Scott Kurowski, *et al.*

Largest prime twins

$33218925 \cdot 2^{169690} \pm 1$ (51090 digits), Daniel Papp and Yves Gallot

Largest prime triplets

$(108748629354 \cdot 4436 \cdot 3251\#(4436 \cdot 3251\# + 1) + 210)\dfrac{4436 \cdot 3251\# - 1}{35} + d,\ d = 7,\ 11,\ 13$ (4135 digits), David Broadhurst

Largest prime quadruplets

$10271674954 \cdot 2999\# + 3461 + d,\ d = 0,\ 2,\ 6,\ 8$ (1284 digits), Michael Bell, Michael Davison, Matt Jack, Ronald Lau, Graeme Leese and Ben Lowing

Largest prime sextuplets

$110282080125 \cdot 700\# + 6005887 + d,\ d = 0,\ 4,\ 6,\ 10,\ 12,\ 16$ (301 digits), Norman Luhn

Largest prime septuplets

$497423806097 \cdot 400\# + 380284918609481 + d,\ d = 0,\ 2,\ 6,\ 8,\ 12,\ 18,\ 20$ (173 digits), Norman Luhn

Titanic prime quintuplets

33267206735443197356651465091271496840063761409618
75870891419221266902387143114128623035533789457982
75752037706697836859777596374478310646941298993893
91530356694718313599175941296123697178103802627343
03909720226166725134828429710196830212759004184171
44903573153569238136751555145176613075183766389335
98297473106916527164722688613058992159137955039225
33169221442419625863660398048641683966468214647409
91150778758840182385169420861094525879108561714153
36162082354196692347160857298741154912372749922393
40413374225364981931382305551444489434539056120741
87346250066683823610863785377926260875147605262304
98279952714131841896881021825644372095309497933764
59534083606134443919612619864247647773483402233339
41876583714212779617441349648225550097247158488548
54923216377958376759658610489971853146340227018039
45222309611809596121641701618545779342557738521950
41741189592730186426255705998985505515170921935737
50106037900368072523069288557346559075362116934081
32546240991220234257748688929696079015903960518025
9427890928720033411211993331280941