Gigantic prime triplets

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You may remember back in December 2002 we defined a *titanic* prime as a non-composite number consisting of at least 1000 decimal digits (Titanic prime quintuplets, M500 **189**, pages 12–13). In the same article I reported the discovery of a large prime quintuplet:

$$31969211688 \prod_{\substack{p\,<\,2400 \\ p \text{ prime}}} p \;+\; 16061 \;+\; d, \qquad d = 0, 2, 6, 8, 12,$$

by Norman Luhn, and I was sufficiently impressed to place all 1034 digits of the first number on the front cover of that issue.

This novel and interesting usage of 'titanic' was introduced in an article by Samuel Yates in 1985. Well, the next power of ten up from one thousand is ten thousand, and, as you can imagine, there is also a technical term for primes of this magnitude. In 1992, when titanic primes were beginning to become commonplace, Yates again realized that a new word was needed; and so he made another definition.

> A *gigantic* prime is defined as a prime number which has at least 10000 decimal digits.

This same definition is used by Chris Caldwell in his database of large primes at http://primes.utm.edu/. You will obviously want to know that the smallest gigantic prime is $10^{9999} + 33603$. This was already 'well known' ever since computer programmers learnt how to do serious arithmetic, but the world had to wait until 2003 for a proof—by Jens Franke, Thorsten Kleinjung and Tobias Wirth.

Now, as I write this, two truly remarkable things have happened.

First, I was most surprised the same **Norman Luhn** wrote to me on 13th October 2008 to report a new gigantic probable prime triplet,

$$2072644824759 \cdot 2^{33333} + d, \qquad d = -1, 1, 5, \qquad (*)$$

at 10047 digits beating his own previous world record of 6223 digits (M500 **220**, cover). The first two members ($d = \pm 1$), each being a factorizable number plus or minus one, are easily proved to be prime by elementary methods. However, the third ($d = 5$) is not of this form, and therefore its primality proof would require a much greater effort. And at 10047 digits, it was at the time not at all clear how this could be done without about 6 months to a year of computing.

Then surprise turned into astonishment when a few weeks later, on 17th November, Norman reported that his third number had been verified in record time by **François Morain** with a new version of his elliptic curve primality prover, FastECPP. Using a cluster of nine AMD Athlon-64 3400^+ processors, Morain achieved the primality proof in a record 111 days of computer time and was able to deliver the required primality certificate in only three weeks, thus confirming (∗) as true prime triplet.

There is an element of history repetition here. A long time ago I reported the 1041-digit *probable* prime triplet

$$2^{3456} + 5661177712051 + d, \qquad d = 0, 2, 6,$$

found in July 1995 (M500 **145**, page 19). If the primes could have been verified quickly, it would have been the first ever titanic example of its kind. However, I had to wait a little longer than Norman—actually more than two years longer—for the primes to be certified by the same François Morain in January 1998 (M500 **161**, page 13).

Norman's triplet is printed full on the front cover of this magazine [reproduced here on page 4].

Some more prime number records, as at 18 December 2008. Notation: $x\# = \prod_{2 \le p \le x,\ p \text{ prime}} p$.

Largest prime: $2^{43112609} - 1$, August 2008, Edson Smith, George Woltman, Scott Kurowski, *et al.* (GIMPS), 12978189 digits.

Largest prime twins: $2003663613 \cdot 2^{195000} \pm 1$, January 2007, Eric Vautier, Dmitri Gribenko, Patrick W. McKibbon, Michaek Kwok, Andrea Pacini and Rytis Slatkevicius, 58711 digits.

Largest prime quadruplets: $4104082046 \cdot 4800\# + 5651 + d$, $d = 0$, 2, 6, 8, April 2005, Norman Luhn, Primo, 2058 digits.

Largest prime quintuplets: $283534892623 \cdot 2500\# + 1091261 + d$, $d = 0$, 2, 6, 8, 12, April 2006, Norman Luhn, 1069 digits.

Largest prime sextuplets: $328481121285 \cdot 1000\# + 16057 + d, d = 0$, 4, 6, 10, 12, 16, January 2006, Norman Luhn, 427 digits.

Largest prime septuplets: $251733155478 \cdot 650\# + 1146779 + d$, $d = 0$, 2, 8, 12, 14, 18, 20, January 2006, Norman Luhn, 282 digits.

Largest prime octuplets: $330846961 \cdot 503\# + 349129635971 + d$, $d = 0$, 2, 6, 8, 12, 18, 20, 26, February 2008, Jens Kruse Andersen, 218 digits.

Largest prime nonuplets: $3336884 \cdot 331\# + 80877403191701 + d, d = 0$, 2, 6, 8, 12, 18, 20, 26, 30, September 2007, Dirk Augustin and Jens Kruse Andersen, 140 digits.

Largest prime decuplets: $24698258 \cdot 239\# + 28606476153371 + d,\ d = 0$, 2, 6, 8, 12, 18, 20, 26, 30, 32, Sept. 2004, Jens Kruse Andersen, 104 digits.

Largest prime 11-tuplets: $24698258 \cdot 239\# + 28606476153371 + d,\ d = 0$, 2, 6, 8, 12, 18, 20, 26, 30, 32, 36, September 2004, Norman Luhn and Jens Kruse Andersen, 104 digits.

Largest prime dodecuplets: $8486221 \cdot 107\# + 4549290807806861 + d,\ d = 0$, 2, 6, 8, 12, 18, 20, 26, 30, 32, 36, 42, May 2006, Dirk Augustin and Jens Kruse Andersen, 50 digits.

Largest prime 14-tuplets: $381955327397348 \cdot 80\# + 18393209 + d,\ d = 0$, 2, 8, 14, 18, 20, 24, 30, 32, 38, 42, 44, 48, 50, December 2007, Norman Luhn, 46 digits. Includes largest prime 13-tuplets.

Largest prime 15-tuplets: $107173714602413868775303366934621 + d, d =$ 0, 2, 6, 8, 12, 18, 20, 26, 30, 32, 36, 42, 48, 50, 56, April 2008, Jens Kruse Andersen, 33 digits.

Largest prime 18-tuplets: $11298510058634407483251313 + d,\ d = 0$, 4, 6, 10, 16, 18, 24, 28, 30, 34, 40, 46, 48, 54, 58, 60, 66, 70, December 2008, Jaroslaw Wroblewski, 26 digits. Includes largest prime 16- and 17-tuplets.

Problem 226.6 – Two bombs

There is a collection of bombs, all of identical construction. Your task is to determine the minimum height from which a bomb must be dropped for the detonation mechanism to work. Great accuracy is not necessary. Measurement to the nearest 10 feet is all that is required. And fortunately there is a convenient very tall building whose floors are spaced ten feet apart.

If you are given just one bomb to test, all you can do is this, starting at $n = 1$. Drop the bomb from floor n and see what happens. If it explodes, report '$10n$ feet'. If not, retrieve the bomb and repeat the test from floor $n + 1$. You may assume that a bomb which survives being dropped will not sustain any damage, and therefore a future test will be valid. On the other hand, once the bomb explodes it cannot be used again.

Now suppose instead you are given *two* test bombs. How can you improve your strategy?

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009339702765920705205561208018366042110825309667327 $+\ d$, $d = 0, 2, 6$, are prime.